Efficient Quasi-Newton Finite Element Formulations for Computational Electromagnetics

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Abstract Fixed-point or Newton-methods are typically employed for the numerical solution of nonlinear systems arising from discretization of nonlinear magnetic field problems. We here discuss an alternative strategy which uses Quasi-Newton updates locally, at every material point, to construct appropriate linearizations of the material behaviour during the nonlinear iterations. The resulting scheme shows similar fast convergence as the Newton-method but, like the fixed-point methods, does not require derivative information of the underlying material law. As a consequence, the method can be used for the efficient solution of models with hysteresis which involve non-smooth material behaviour.

1 Introduction

A major challenge in computational electromagnetics is the efficient and robust incorporation of vector-based magnetic material models into the finite element method (FEM), which can also account for hysteretic effects. The Jiles-Atherton (JA) model has been implemented by various authors using a differential permeability approach described for the reduced scalar potential formulation using the Newton method [5]. A similar approach to incorporate the JA model has also been done for the vector potential formulation in [4]. However, the JA model is derived from a differential equation describing the differential permeability/reluctance and therefore provides an elegant way to use it in the finite element formulation. This is not the case for the energy-based hysteresis models used in this work to describe hysteretic effects. Nevertheless, there are approaches in the literature that use energy-based hysteresis models and use them in the FEM for vector and reduced scalar potential formulations [6], [7], [10]. In addition, other models have been incorporated into the FEM, such as the inverse G model in [3] and the parametric algebraic model (PAM) in [8]. In [10] the convergence of the Newton method is analysed, and the authors employ automatic differentiation to evaluate the differential reluctivity tensor.

2 Energy based vector hysteresis

The energy-based hysteresis operator has been derived in terms of the Helmholtz and Gibbs free energies in a thermodynamically consistent manner [9]. The pinning effect of the magnetic material causes the dissipation of the system. A state transition can be linked to an incremental energy minimisation used in e.g. elasto-plasticity theory. The numerical solution of this minimisation problem is obtained by the efficient Newton-Raphson method. To account for the correct physical prediction of rotational losses, the pinning force is made dependent on the saturation state so that the irreversibility vanishes. We define the power law function

$$\xi(\boldsymbol{J}) = 1 - \left(\frac{|\boldsymbol{J}|}{J_{\mathrm{sat}}}\right)^{\alpha} \tag{1}$$

with $\boldsymbol{J}$ the magnetic polarization and α an appropriate exponent being identified based on measurements. In general, ξ is defined so that it has a maximum value at zero magnetic polarisation and vanishes at saturation polarisation, as shown in Fig. 1.

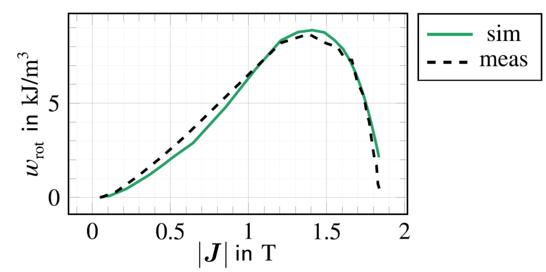

Fig. 1. Comparison of measured and simulated rotational loss density of an electric steel sheet [9].

2 Quasi-Newton formulation

The general constitutive law of magnetics between the magnetic flux density $\boldsymbol{B}$ and field intensity $\boldsymbol{H}$ in operator notation reads as

$$\boldsymbol{B} = \mathcal{B}(\boldsymbol{H})\,. \tag{2}$$

Newton's scheme for the partial differential equation for the magnetic scalar potential ϕ can be written in its weak formulation with the test function ϕ' by [1]

$$\begin{aligned} \int_\Omega \nabla\phi' \cdot \frac{\partial\mathcal{B}(\boldsymbol{H})}{\partial\boldsymbol{H}}\bigg|_{\boldsymbol{H}_k} \cdot \nabla s \,\mathrm{d}\Omega' &= \int_\Omega \nabla\varphi' \cdot \mathcal{B}(\boldsymbol{H}_k)\,\mathrm{d}\Omega' \\ \phi_{k+1} &= \phi_k + \eta s\,. \end{aligned} \tag{3}$$

The Jacobian $\partial\mathcal{B}(\boldsymbol{H})/\partial\boldsymbol{H}$ can be interpreted as a differential permeability tensor. The Davidon-Fletcher-Powell

(DFP) formula is rank 2, preserves symmetry as well as positive definiteness, and is computed by [1]

$$\frac{\partial \mathcal{B}^k}{\partial \boldsymbol{H}} = \left(\boldsymbol{I} - \frac{\Delta\mathcal{B}\Delta\boldsymbol{H}^T}{\Delta\mathcal{B}^T\Delta\boldsymbol{H}}\right)\frac{\partial \mathcal{B}^{k-1}}{\partial \boldsymbol{H}} \left(\boldsymbol{I} - \frac{\Delta\boldsymbol{H}\Delta\mathcal{B}^T}{\Delta\mathcal{B}^T\Delta\boldsymbol{H}}\right) + \frac{\Delta\mathcal{B}\Delta\mathcal{B}^T}{\Delta\mathcal{B}^T\Delta\boldsymbol{H}}$$

with the unity tensor $\boldsymbol{I}$. For this approach, we can derive a full convergence analysis for the iterations using local Quasi-Newton updates based on a low order finite element approximation of the scalar potential formulation of magnetostatics [2]. However, the main arguments also apply to vector potential formulations and higher order approximations. After appropriate time discretisation, the algorithms may also be useful for solving nonlinear eddy current problems, with applications to induction heating and the simulation of superconductivity.

3 Numerical results

Here we consider the excitation by a sequence of load currents taken from test case 2 of TEAM problem 32. We perform the calculation once for the anhysteretic case (classical nonlinear BH curve) and once for the hysteretic case. The results of our calculations are summarised in Tab. I. Compared to the simulations for the classical

	Average iterations	
	Nonlineare case	Hysteretic case
1477	3.5	7.2
5789	3.6	7.3
22921	3.6	7.4
91217	3.6	7.4

TABLE I
AVERAGE ITERATION NUMBERS FOR SOLUTION FOR $N = 402$ TIME STEPS (TEST CASE 2 OF TEAM PROBLEM 32).

nonlinear (anhysteretic) case, the iteration numbers are approximately doubled for all methods investigated. In Fig. 2 we compare the hysteretic and anhysteretic models with the measurements. The plot shows the relevant component of the magnetic induction field.

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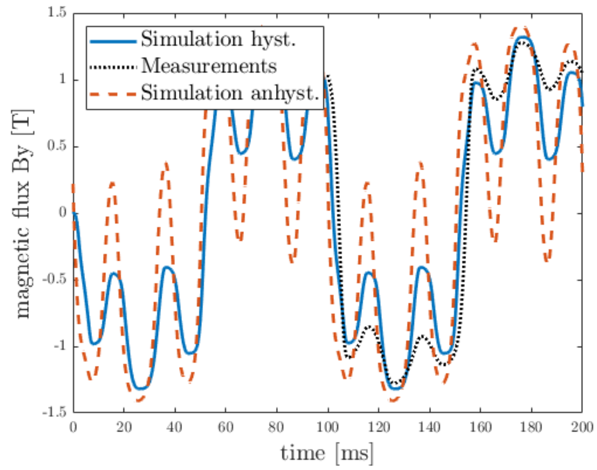

Fig. 2. Comparison of the hysteretic and anhysteretic model with the measurements.

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